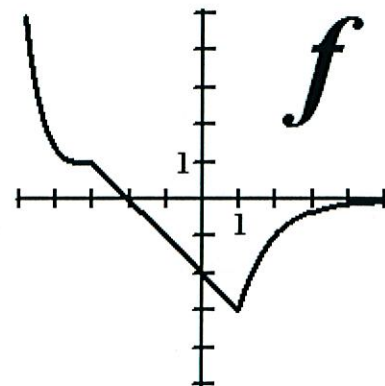


Let  $g(x) = \int_0^x f(t) dt$ , where  $f$  is the function whose graph is shown on the right.

SCORE: \_\_\_\_ / 5 PTS

- [a] Find  $g'(2)$ . Explain your answer very briefly.

$$\underbrace{g'(2) = f(2)}_{\textcircled{\frac{1}{2}}} = \underbrace{-1}_{\textcircled{\frac{1}{2}}}$$



- [b] Find all intervals over which  $g$  is increasing. Explain your answer very briefly.

$$\underbrace{g'(x) = f(x) > 0}_{\textcircled{1}} \text{ on } (-\infty, -2) \text{ or } (-5, -2) \quad \left. \begin{array}{l} \textcircled{1} \\ \text{EITHER} \\ \text{IS OK} \end{array} \right\}$$

- [c] Find all intervals over which  $g$  is concave up. Explain your answer very briefly.

$$\underbrace{g'(x) = f(x) \text{ INCREASING}}_{\textcircled{1}} \text{ on } (1, \infty) \text{ or } (1, 5) \quad \left. \begin{array}{l} \textcircled{1} \\ \text{EITHER} \\ \text{IS OK} \end{array} \right\}$$

A new office building is renting office space month-by-month for  $R(x)$  dollars per square meter if the tenant

SCORE: \_\_\_\_ / 3 PTS

rents  $x$  square meters. What is the meaning of the equation  $\int_{2000}^{2200} R(x) dx = 3000$  in this situation ?

**NOTES:** Your answer must use all three numbers from the equation, along with correct units.

Your answer should NOT use "x", " $R(x)$ ", "integral", "antiderivative", "rate of change" or "derivative".

IF YOU ENLARGE YOUR OFFICE FROM  $2000 \text{ m}^2$  TO  $2200 \text{ m}^2$ ,  
YOUR RENT WILL INCREASE BY \$3000

GRADED  
BY ME

AJ was looking at BJ's homework.

SCORE: \_\_\_\_ / 3 PTS

AJ said that BJ's work (shown below) was wrong, but BJ insisted that it was correct. Who was right, and why ?

$$\int_0^{\pi} \sec^2 \theta d\theta = \tan \theta \Big|_0^{\pi} = \tan \pi - \tan 0 = 0 - 0 = 0$$

AJ -  $\sec^2 \theta$  IS NOT CONTINUOUS ON  $[0, \pi]$  @  $\theta = \frac{\pi}{2}$

GRADED BY ME

If  $p(x) = \int_{e^{4x}}^{\cosh^{-1} x} \cos^8 t \, dt$ , find  $p'(x)$ .

SCORE: \_\_\_\_ / 4 PTS

$$\begin{aligned} & \frac{d}{dx} \left[ \int_{e^{4x}}^{\cosh^{-1} x} \cos^8 t \, dt + \int_0^{\cosh^{-1} x} \cos^8 t \, dt \right] \\ &= \frac{d}{dx} \left[ - \int_0^{e^{4x}} \cos^8 t \, dt + \int_0^{\cosh^{-1} x} \cos^8 t \, dt \right] \quad (1) \\ &= - \frac{d}{d(e^{4x})} \int_0^{e^{4x}} \cos^8 t \, dt \cdot \frac{d(e^{4x})}{dx} + \frac{d}{d(\cosh^{-1} x)} \int_0^{\cosh^{-1} x} \cos^8 t \, dt \cdot \frac{d(\cosh^{-1} x)}{dx} \\ &= - \frac{1}{4e^{4x}} \cos^8 e^{4x} + \frac{1}{\sqrt{x^2-1}} \cos^8 \cosh^{-1} x \quad (1) \end{aligned}$$

Evaluate the following integrals.

ALL PARTS WORTH  $\frac{1}{2}$  POINT  
EXCEPT THOSE  
MARKED  
EXPLICITLY

SCORE: \_\_\_\_ / 10 PTS

[a]  $\int \frac{(3+y)^2}{\sqrt[3]{y}} dy$

$$\begin{aligned} &= \int \frac{9+6y+y^2}{y^{\frac{1}{3}}} dy \\ &= \int (9y^{-\frac{1}{3}} + 6y^{\frac{2}{3}} + y^{\frac{5}{3}}) dy \quad (1) \\ &= \frac{27}{2} y^{\frac{2}{3}} + \frac{18}{5} y^{\frac{5}{3}} + \frac{3}{8} y^{\frac{8}{3}} + C \end{aligned}$$

[b]  $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{(\pi - \arctan 2t)^2}{1+4t^2} dt$

$$\begin{aligned} & u = \pi - \arctan 2t \\ & du = - \frac{2}{1+4t^2} dt \\ & -\frac{1}{2} du = \frac{1}{1+4t^2} dt \end{aligned}$$

$$\begin{aligned} t = -\frac{1}{2} &\rightarrow u = \pi - \arctan(-1) \\ &= \pi - (-\frac{\pi}{4}) = \frac{5\pi}{4} \\ t = \frac{1}{2} &\rightarrow u = \pi - \arctan 1 \\ &= \pi - \frac{\pi}{4} = \frac{3\pi}{4} \end{aligned}$$

$$\begin{aligned} & \int_{\frac{5\pi}{4}}^{\frac{3\pi}{4}} -\frac{1}{2} u^2 du = -\frac{1}{6} u^3 \Big|_{\frac{5\pi}{4}}^{\frac{3\pi}{4}} \quad (1) \\ &= -\frac{1}{6} \left( \left( \frac{3\pi}{4} \right)^3 - \left( \frac{5\pi}{4} \right)^3 \right) \\ &= -\frac{1}{6} \left( \frac{27\pi^3}{64} - \frac{125\pi^3}{64} \right) \\ &= -\frac{1}{6} \cdot \frac{-98\pi^3}{64} \\ &= \frac{49\pi^3}{192} \quad (1) \end{aligned}$$

[c]  $\int_{-1}^1 \frac{7x^3}{(3-2x^6)^5} dx$

$f(x)$  IS CONTINUOUS ON  $[-1, 1]$

$$f(-x) = \frac{7(-x)^3}{(3-2(-x)^6)^5} = -\frac{7x^3}{(3-2x^6)^5} = -f(x) \quad (1)$$

SO  $f(x)$  IS ODD ON  $[-1, 1]$

SO  $\int_{-1}^1 f(x) dx = 0$

MUST  
HAVE  
"du"